Enrollment No:-__________ Exam Seat No:-________

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 4SC04MTC2 **Subject Name: Linear Algebra-II**

Course Name: B.Sc. Date: 21/5/2015

Semester: IV Marks: 70

Time: 10:30 TO 01:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION – I

Q-1 (A) What is angle between $u=e_1$ and $v=e_1+e_2$ in R^2. [01]

(B) Find $cos\theta$ where θ is angle between $f(t)=t$ and $g(t)=t^2$ in C[0,1] further $\langle f,g\rangle=\int_0^1 f(t)g(t)dt$. [02]

(C) Define orthogonal linear transformation. [02]

(D) If W is subspace of vector space V .then show that $W^{\perp}\cap W = \{0\}$. [02]

Q-2 (A) State and prove Riesz-representation theorem. [07]

(B) Apply Gram-schmidth process to obtain orthonormal set $\{(1,1,1,1),(0,2,0,2),(-1,1,3,-1)\}$ in R^4. [07]

OR

Q-2 (A) Using gram-schmidth process obtain orthonaormal set for$\{1, t, t^2\}$ of with inner product $\langle p,q\rangle=\int_0^1 p(t)q(t)dt$. [07]

(B) If W is subspace of inner product space V then show that V=W$\oplus$ $W^{\perp}$. [07]

Q-3 (A) Show any subspace W of R^n is the set of solution of homogenous system of linear equations. [07]

(B) Let T:V→V be linear then the following are equivalent. [07]

(1) T is orthogonal.
(2) $\|Tx\| = \|x\|$,for all x$\in V$.
(3) T takes an orthonarmal basis to orthonormal basis.

OR

21-5

Q-3 (A) If is any map such that [07]

(1) $f(0)=0$

(2) $\|f(x)-f(y)\| = \|x-y\|$,

then show that f is an orthogonal map.

(B) With usual notation and figure show that [07]

$R_\theta=\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$ and $\rho_\theta=\begin{pmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{pmatrix}$.

Section-II

Q-4 (A) For what values of r, $det(\alpha A) = \alpha^r det$ A. [01]

(B) det (A + B) = det A + det B. Is this statement true or false? Justify your answer. [02]

(C) Define eigen –value and eigen vector. [02]

(D) All characteristic roots of symmetric matrix are real. Is this statement true or false? [02]

Q-5 (A) Show that parallelogram is rhombus if and only if the diagonals are perpendicular to each other. [05]

(B) Show that medians of a triangle are con current. [05]

(C) Show that the angle inscribed by semicircle is a right angle. [04]

OR

Q-5 (A) For a linear system Ax=b in usual notation show that $x_1 = \frac{\det(b,A_2,A_3)}{\det A}$. [05]

(B) Solve by Cramer's rule [05]

$x + y = 0$

$y + z = 1$

$z + x = -1$

(C) If $A=\begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, then compute det A using column vectors and inner product. [04]

Q-6 (A) If $x=(x_1,x_2,x_3)$ and $y=(y_1,y_2,y_3)$ then show that $x \times y = \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}$. [07]

(B) Find the eigen values and eigen vectors of $\begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ [07]

OR

Q-6 (A) State and Prove Caley-Hamilton theorem. [07]

(B) Reduce the equation $2x^2 - 72xy + 23y^2 + 140x - 20y + 50 = 0$ in to standard form. [07]